

PAPER CODE - 8195

12<sup>th</sup> CLASS - 1<sup>st</sup> Annual 2023

MATHEMATICS  
GROUP : FIRST

DGK-12-1-23

OBJECTIVE

TIME: 30 MINUTES

MARKS: 20

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

QUESTION NO. 1

- 1  $\int e^{-x} (\cos x - \sin x) dx = \dots\dots\dots$   
(A)  $-e^{-x} \sin x + c$  (B)  $e^{-x} \sin x + c$  (C)  $e^x \cos x$  (D)  $-e^x \cos x + c$
- 2 The order of differential equation  $x^2 \frac{d^2y}{dx^2} + \frac{dy}{dx} + 2x = 0$  is  
(A) 1 (B) 2 (C) 3 (D) 4
- 3 Vertical line passes through (5,4) is  
(A)  $y = 4$  (B)  $x = 5$  (C)  $y = 5$  (D)  $y = -4$
- 4 Slope of line perpendicular to  $3x - 4y + 5 = 0$  is  
(A)  $-4/3$  (B)  $-3/4$  (C)  $3/4$  (D)  $4/3$
- 5 Coordinate of mid-point of A (-1, 4) and B(6, 2) is  $\dots\dots\dots$   
(A) (-7, 2) (B) (7, -2) (C)  $(5/2, 3)$  (D)  $(5/2, -5/2)$
- 6 Graph of  $4y \geq 5$  will be  $\dots\dots\dots$  half plane  
(A) lower (B) right (C) upper (D) left
- 7 Directrix of  $y^2 = 8x$  is  
(A)  $x + 2 = 0$  (B)  $x - 2 = 0$  (C)  $y + 2 = 0$  (D)  $y - 2 = 0$
- 8 Vertices of the ellipse  $\frac{x^2}{16} + \frac{y^2}{25} = 1$  are  $\dots\dots\dots$   
(A) (0,  $\pm 4$ ) (B) ( $\pm 4, 0$ ) (C) ( $\pm 5, 0$ ) (D) (0,  $\pm 5$ )
- 9 The center of circle  $x^2 + y^2 - 6x + 4y + 13 = 0$  is  
(A) (3, -2) (B) (-3, 2) (C) (-3, -2) (D) (3, 2)
- 10 An angle in the semi-circle is of measure  $\dots\dots\dots$   
(A)  $30^\circ$  (B)  $90^\circ$  (C)  $45^\circ$  (D)  $60^\circ$
- 11  $\begin{bmatrix} k & i & j \end{bmatrix} = \dots\dots\dots$   
(A) 1 (B) -1 (C) 0 (D) 3
- 12 If  $\underline{U} = i + \alpha j - k$  and  $\underline{V} = 2i + j + k$  are perpendicular then  $\alpha = \dots\dots\dots$   
(A) 1 (B) 2 (C) -1 (D) 0
- 13  $f(x) = x \quad \forall x \in \mathbb{R}$  is called  $\dots\dots\dots$   
(A) Constant function (B) Identity function (C) Non-linear function (D) Trigonometric function
- 14  $\lim_{x \rightarrow 0} (1-x)^{1/x} = \dots\dots\dots$   
(A)  $e^x$  (B)  $\infty$  (C)  $e^{1/x}$  (D)  $e^{-1}$
- 15  $\frac{d}{dx} (\tan x) = \dots\dots\dots$   
(A)  $\ln \cos x$  (B)  $-\ln \cos x$  (C)  $\sec^2 x$  (D)  $-\sec^2 x$
- 16 If  $f(x) = \sin x$  then  $f'(\frac{\pi}{2}) = \dots\dots\dots$   
(A) 0 (B) 1 (C) 2 (D) -1
- 17  $\frac{d}{dx} (\cosh 2x) = \dots\dots\dots$   
(A)  $\cosh 2x$  (B)  $2 \cosh 2x$  (C)  $2 \sinh 2x$  (D)  $\sinh 2x$
- 18 For a stationary point of function we have  $f'(x) = \dots\dots\dots$   
(A) 0 (B) Positive (C) Negative (D)  $\infty$
- 19 If  $v = x^3$  then differential of v is  
(A)  $3x^2$  (B)  $3x^2 dv$  (C)  $x^3 dx$  (D)  $3x^2 dx$
- 20  $\int \frac{\sec^2 x}{\tan x} dx = \dots\dots\dots$   
(A)  $\tan x + c$  (B)  $-\cot x + c$  (C)  $\ln(\tan x) + c$  (D)  $\sec x + c$

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QUESTION NO. 2 Write short answers any Eight (8) of the following

16

i	Express the area A of a circle as a function of its circumference C.
ii	For any real valued function of $f(x) = 2x + 1$ , find $f \circ f(x)$ .
iii	Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin \theta}$
iv	Differentiate $(x - 5)(3 - x)$ w.r.t $x$
v	Find $\frac{dy}{dx}$ if $xy + y^2 = 2$
vi	Find $\frac{dy}{dx}$ if $y = x \cos y$
vii	Find $f'(x)$ if $f(x) = e^x(1 + \ln x)$
viii	Find $y_2$ if $x^2 + y^2 = a^2$
ix	Apply Maclaurin series expansion to prove that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
x	Find the extreme values for the function $f(x) = 5x^2 - 6x + 2$
xi	Define convex region.
xii	Graph the solution set of the inequality $5x - 4y \leq 20$

QUESTION NO. 3 Write short answers any Eight (8) of the following

16

i	Evaluate $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$
ii	Evaluate $\int \frac{adt}{2\sqrt{at+b}}$
iii	Find $\int x \ln x dx$
iv	Evaluate the definite integral $\int_{-6}^2 \sqrt{3-x} dx$
v	Evaluate $\int \frac{2x}{x^2-a^2} dx$ , $x > a$
vi	Evaluate $\int (x+1)(x-3) dx$
vii	Evaluate $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ , $x > 0$
viii	Define equal Vectors.
ix	Find the unit vector in the direction of the vector $\underline{v} = 2\underline{i} + 6\underline{j}$
x	Let $\vec{A} = (2,5)$ , $B(-1,1)$ Find $\overline{AB}$
xi	Write two properties of Dot Product.
xii	Define cross product of two vectors and give its geometrical meanings.

QUESTION NO. 4 Write short answers any Nine (9) of the following

18

i	The points A (-5,-2) and B(5,-4) are ends of diameter of Circle, Find the Center and radius of Circle.
ii	The coordinates of P are (-6, 9), the axes are translated through point O'(-3,2), Find coordinate of P referred to new axes.
iii	By means of slopes, show that (4,-5), (7,5) and (10, 15) lie on same line.
iv	Find equation of line whose x-intercept is -3, y-intercept is 4.
v	Convert $15y - 8x + 3 = 0$ into normal and slope intercept form.
vi	Check whether the lines $4x - 3y - 8 = 0$ , $3x - 4y - 6 = 0$ and $x - y - 2 = 0$ are concurrent.
vii	Find lines represented by $6x^2 - 19xy + 15y^2 = 0$
viii	Find centre and radius of circle $5x^2 + 5y^2 + 24x + 36y + 10 = 0$
ix	Find equation of circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$
x	Write equation of tangent to $3x^2 + 3y^2 + 5x - 13y + 2 = 0$ at $\left(1, \frac{10}{3}\right)$
xi	Find focus and vertex of parabola $y^2 = -8(x - 3)$
xii	Find equation of ellipse having centre (0, 0), focus at (0, -3) and one vertex at (0, 4)
xiii	Find eccentricity and vertices of hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(P.T.O)

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DGR-12-1-23

SECTION-II

Note: Attempt any Three questions from this section

10 x 3 = 30

Q.5-(A)	Find the values $m$ and $n$ , so that the given function is continuous at $x = 3$ $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$
(B)	If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ show that $2x \frac{dy}{dx} + y = 2\sqrt{x}$
Q.6-(A)	Evaluate the indefinite integral $\int \sqrt{x^2 - a^2} dx$
(B)	Find the equation of the medians of triangle whose vertices are $A(-3,2)$ , $B(5,4)$ and $C(3,-8)$
Q.7-(A)	Evaluate $\int_0^{\pi/4} (1 + \cos^2 \theta) \tan^2 \theta d\theta$
(B)	Maximize $f(x, y) = x + 3y$ ; subject to the constraints $2x + 5y \leq 30$ $5x + 4y \leq 20$ $x \geq 0, y \geq 0$
Q.8-(A)	Find $f'(x)$ if $f(x) = \sqrt{\ln(e^{2x} + e^{-2x})}$
(B)	Write an equation of the circle that passes through the points $A(4,5)$ , $B(-4, -3)$ , $C(8, -3)$
Q.9-(A)	Find the focus, vertex and directrix of the parabola $x + 8 - y^2 + 2y = 0$
(B)	Prove that angle in a semi circles is a right angle.

NOTE: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question.

QUESTION NO. 1

- 1  $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] =$   
 (A)  $\frac{f(x)g'(x) - f'(x)g(x)}{[g(x)]^2}$  (B)  $\frac{f'(x)g(x) - g'(x)f(x)}{[f(x)]^2}$  (C)  $\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$  (D)  $\frac{g'(x)f'(x) - f(x)g(x)}{[g(x)]^2}$
- 2  $\frac{1}{1+x^2}$  is derivation of  
 (A)  $\sin^{-1} x$  (B)  $\sec^{-1} x$  (C)  $\tan^{-1} x$  (D)  $\cot^{-1} x$
- 3  $\int \ln x \, dx$  is equal to  
 (A)  $x - x \ln x + c$  (B)  $x \ln x + x + c$  (C)  $\frac{1}{x} \ln x + c$  (D)  $x \ln -x + c$
- 4  $\int_1^2 (x^2 + 1) \, dx =$   
 (A)  $\frac{3}{10}$  (B) 2 (C)  $\frac{10}{3}$  (D) 0
- 5  $\int a^x \, dx =$   
 (A)  $\frac{a^x}{\ln a} + c$  (B)  $\frac{\ln a}{a^x} + c$  (C)  $\frac{1}{a^x \ln a} + c$  (D)  $a^x \ln a + c$
- 6 The solution of differential equation  $\frac{dy}{dx} = -y$  is  
 (A)  $y = x e^{-x}$  (B)  $y = c e^{-x}$  (C)  $y = e^x$  (D)  $y = c e^x$
- 7 The distance between the points (0, 0) and (1, 2) is  
 (A) 0 (B) 1 (C) 2 (D)  $\sqrt{5}$
- 8 A linear equation in two variables represents  
 (A) circle (B) ellipse (C) hyperbola (D) straight line
- 9 The slope- intercept form of equation of line is  
 (A)  $y = \frac{1}{m} x - c$  (B)  $y = mx + c$  (C)  $y = cx + m$  (D)  $y = cx - m$
- 10 Bisectors of angles of a triangle are  
 (A) Parallel (B) Perpendicular (C) Concurrent (D) Non-concurrent
- 11 The feasible solution which maximizes or minimizes the objective function is called  
 (A) Exact solution (B) Final solution (C) Optimal solution (D) Objective solution
- 12 Equation of circle with centre at origin and radius  $\sqrt{5}$  is  
 (A)  $x^2 + y^2 = \sqrt{5}$  (B)  $x^2 + y^2 = 5$  (C)  $x^2 + y^2 = 25$  (D)  $(x - 3)^2 + y^2 = 5$
- 13 The parabola  $y^2 = 4ax$ ,  $a > 0$  opens  
 (A) Right (B) Left (C) Upward (D) Downward
- 14 In an ellipse, the foci lie on  
 (A) Major axis (B) Minor axis (C) Directrix (D) Z-axis
- 15 If  $\vec{F} = 4\vec{i} + 3\vec{j} + 5\vec{k}$  and  $\vec{d} = -\vec{i} + 3\vec{j} + 8\vec{k}$ , then work done is  
 (A) 30 unit (B) 45 unit (C) 53 unit (D) 47 unit
- 16 If  $\underline{U}$ ,  $\underline{V}$  and  $\underline{W}$  are coterminal edges of a tetrahedron, then its volume is  
 (A)  $[\underline{U} \underline{V} \underline{W}]$  (B)  $\frac{1}{3} [\underline{U} \underline{V} \underline{W}]$  (C)  $\frac{1}{6} [\underline{U} \underline{V} \underline{W}]$  (D)  $\frac{1}{9} [\underline{U} \underline{V} \underline{W}]$
- 17 If  $f(x) = x^2$ , then range of f is  
 (A) All non-negative real numbers (B) Rational numbers (C) Integers (D) Irrational numbers
- 18  $\lim_{\theta \rightarrow 0} \frac{\sin 7\theta}{\theta} =$   
 (A) 7 (B)  $\frac{1}{7}$  (C) 1 (D)  $\frac{2}{7}$
- 19  $\frac{d}{dx} (x^{an}) = 0$   
 (A)  $-anx^{an-1}$  (B)  $anx^{an-1}$  (C)  $(an-1)x^{an-1}$  (D)  $\frac{x^{an+1}}{an+1}$
- 20 If  $y = \frac{1}{x^2}$ , then  $\frac{dy}{dx}$  at  $x = -1$  is  
 (A) 2 (B) 3 (C)  $\frac{1}{3}$  (D) 4

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QUESTION NO. 2 Write short answers any Eight (8) of the following

16

i	Prove the identity $\operatorname{sech}^2 x = 1 - \tanh^2 x$
ii	Evaluate $\lim_{x \rightarrow \infty} \left( \frac{x}{1+x} \right)^x$
iii	If $f(x) = \begin{cases} x+2, & x \leq -1 \\ c+2, & x > -1 \end{cases}$ , Find C so that $\lim_{x \rightarrow -1} f(x)$ exists
iv	Differentiate w.r.t 'x' $(\sqrt{x} - \frac{1}{\sqrt{x}})^2$
v	Find $\frac{dy}{dx}$ , if $x = \theta + \frac{1}{\theta}$ and $y = \theta + 1$
vi	Differentiate w.r.t 'x' $\cos \sqrt{x} + \sqrt{\sin x}$
vii	Find $f'(x)$ if $f(x) = \frac{e^x}{e^{-x} + 1}$
viii	Find $y_2$ if $x = at^2$ , $y = bt^4$
ix	Apply Maclaurin series expansion to prove $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \dots$
x	Find two positive integers whose sum is 30 and their product will be maximum.
xi	Graph the solution region of linear inequality $3x - 2y \geq 6$
xii	Graph the linear inequality $2x \geq -3$ in $xy$ - plane.

QUESTION NO. 3 Write short answers any Eight (8) of the following

16

i	Find $\int x \cos x \, dx$
ii	Evaluate $\int x^2 \tan^{-1} x \, dx$
iii	Evaluate $\int_0^{\frac{\pi}{3}} \cos^2 \theta \sin \theta \, d\theta$
iv	Evaluate $\int_1^e x \ln x \, dx$
v	Find area between the x-axis and the curve $y = 4x - x^2$
vi	Solve $\frac{dy}{dx} = \frac{y^2 + 1}{e^{-x}}$
vii	Solve the differential equation $\sec x + \tan y \frac{dy}{dx} = 0$
viii	If $\vec{AB} = \vec{CD}$ . Find coordinates of the point A when points B, C, D are (1, 2), (-2, 5), (4, 11) respectively.
ix	Prove $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$
x	Find a vector whose magnitude is 4 and is parallel to $2\hat{i} - 3\hat{j} + 6\hat{k}$
xi	Show that the components of a vector are projections of that vectors along $\hat{i}$ , $\hat{j}$ and $\hat{k}$ respectively.
xii	Show that the vectors $3\hat{i} - 2\hat{j} + \hat{k}$ , $\hat{i} - 3\hat{j} + 5\hat{k}$ and $2\hat{i} - \hat{j} - 4\hat{k}$ form a right angle triangle.

QUESTION NO. 4 Write short answers any Nine (9) of the following

18

i	Show that for the points A (3, 1), B (-2, -3) and C (2, 2), $ \vec{AB}  =  \vec{BC} $
ii	Find the point that divide the join of A (-6, 3) and B (5, -2) in the ratio 2 : 3 internally.
iii	Find the slope and inclination of line joining the points (4, 6); (4, 8)
iv	Find an equation of line with x-intercept : -9 and slope : -4
v	Find the area of triangle whose vertices are A (2, 3), B (-1, 1) and C (4, -5)
vi	Find the lines represented by the equation $2x^2 + 3xy - 5y^2 = 0$
vii	Find an equation of the line through (11, -5) and parallel to a line with slope -24
viii	Find an equation of circle with centre (-3, 5) and radius 7
ix	Find centre and radius of circle $x^2 + y^2 - 6x + 4y + 13 = 0$
x	Check the position of the point (5, 6) w.r.t circle $x^2 + y^2 = 81$
xi	Find an equation of parabola with focus (-3, 1) and directrix $x = 3$
xii	Find centre and foci of the ellipse $x^2 + 4y^2 = 16$
xiii	Find foci and vertices of hyperbola $\frac{y^2}{4} - x^2 = 1$

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(P.T.O)

**SECTION-II**

DGK-12-2-23

Note: Attempt any Three questions from this section

10 x 3 = 30

Q.5-(A)	Find the values $m$ and $n$ so that the given function $f(x)$ is continuous at $x = 3$ $f(x) = \begin{cases} mx & \text{if } x < 3 \\ n & \text{if } x = 3 \\ -2x + 9 & \text{if } x > 3 \end{cases}$
(B)	If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$ show that $2x \frac{dy}{dx} + y = 2\sqrt{x}$
Q.6-(A)	Evaluate $\int e^{2x} \cos 3x \, dx$
(B)	Find an equation of the line through $(5, -8)$ and perpendicular to the join of $A(-15, -8)$ , $B(10,7)$
Q.7-(A)	Find the area between the x-axis and the curve $y = \sqrt{2ax - x^2}$ , where $a > 0$
(B)	Maximize $f(x,y) = x + 3y$ subject to the constraints $2x + 5y \leq 30$ ; $5x + 4y \leq 20$ ; $x \geq 0$ ; $y \geq 0$
Q.8-(A)	Find $y_4$ if $y = \cos^3 x$
(B)	Find equation of circle passing through $A(3, -1)$ , $B(0, 1)$ and having centre at $4x - 3y - 3 = 0$
Q.9-(A)	Find the centre, foci eccentricity, vertices and equation of directrices of $\frac{(x-1)^2}{2} - \frac{(y-1)^2}{9} = 1$
(B)	Prove that $C = a \cos B + b \cos A$ .